

AD-A071 611

MICHIGAN UNIV ANN ARBOR HUMAN PERFORMANCE CENTER  
CONSTRAINED NONMETRIC MULTIDIMENSIONAL SCALING.(U)  
JUN 79 E NOMA, J JOHNSON

F/G 12/1

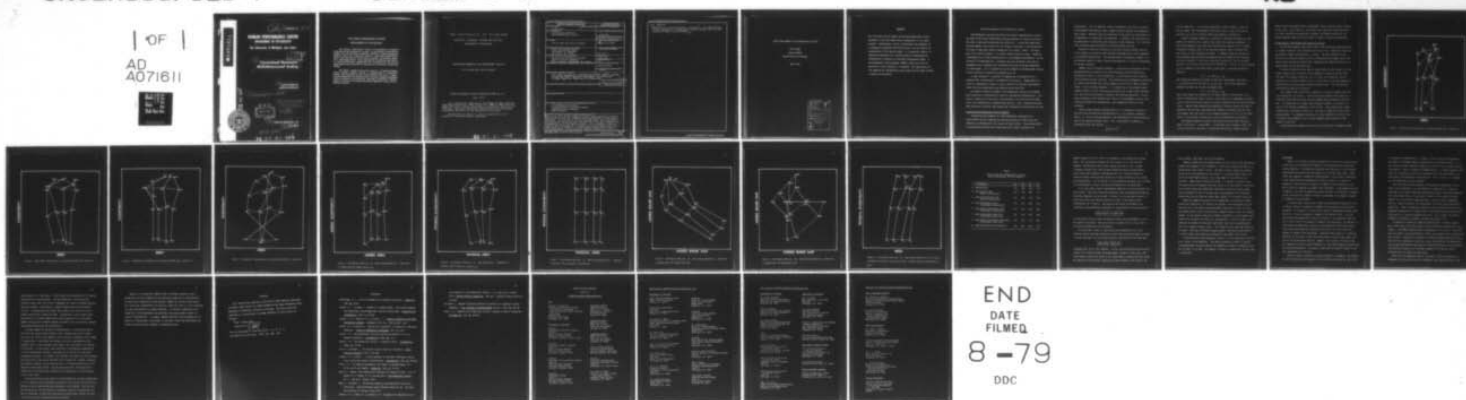
UNCLASSIFIED

014523-5-T

N00014-76-C-0648

NL

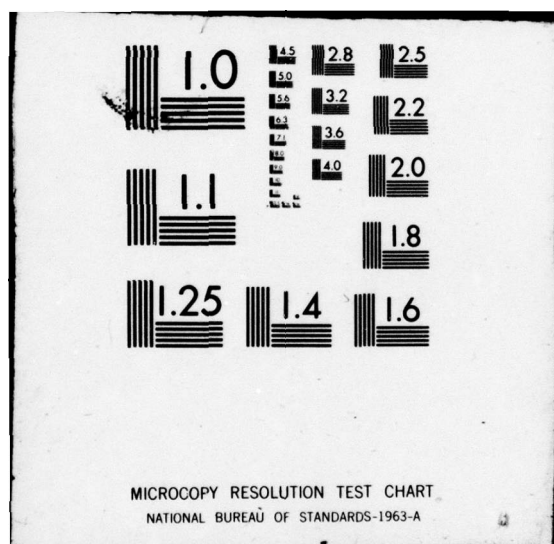
1 OF 1  
AD  
A071611



END  
DATE  
FILMED

8-79

DDC



AD A 071611

(12) (14) 014523-5-T, TR-62

# HUMAN PERFORMANCE CENTER

## DEPARTMENT OF PSYCHOLOGY

The University of Michigan, Ann Arbor

LEVEL #

(6)

**Constrained Nonmetric  
Multidimensional Scaling**

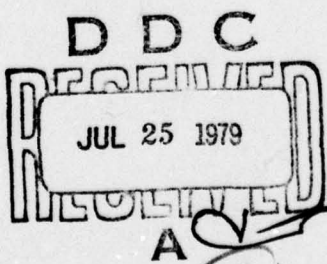
(10)

ELLIOT/NOMA &  
JANICE/JOHNSON

(15)

NOO 14-76-C-0648

DDC FILE COPY



DISTRIBUTION STATEMENT A  
Approved for public release;  
Distribution Unlimited

(12) 35p.

(9)

Technical Report, No. 62

(11) June 1979

402 524

LB

79 07 23 097

## THE HUMAN PERFORMANCE CENTER

### DEPARTMENT OF PSYCHOLOGY

The Human Performance Center is a federation of research programs whose emphasis is on man as a processor of information. Topics under study include perception, attention, verbal learning and behavior, short- and long-term memory, choice and decision processes, and learning and performance in simple and complex skills. The integrating concept is the quantitative description, and theory, of man's performance capabilities and limitations and the ways in which these may be modified by learning, by instruction, and by task design.

The Center issues two series of reports. A Technical Report series includes original reports of experimental or theoretical studies, and integrative reviews of the scientific literature. A Memorandum Report series includes printed versions of papers presented orally at scientific or professional meetings or symposia, methodological notes and documentary materials, apparatus notes, and exploratory studies.



THE UNIVERSITY OF MICHIGAN

COLLEGE OF LITERATURE, SCIENCE AND THE ARTS

DEPARTMENT OF PSYCHOLOGY

CONSTRAINED NONMETRIC MULTIDIMENSIONAL SCALING

Elliot Noma AND Janice Johnson

HUMAN PERFORMANCE CENTER TECHNICAL REPORT NO. 62

June, 1979

This research was supported by the Office of Naval Research, Department of Defense, under Contract No. N0014-76-0648 with the Human Performance Center, Department of Psychology, University of Michigan. The senior author was supported by a training grant from NIGMS (GM-01231) to the University of Michigan.

Reproduction in whole or in part is permitted for any purpose of the United States Government.

79 07 23 097

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 014523-5-T	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Constrained Nonmetric Multidimensional Scaling		5. TYPE OF REPORT & PERIOD COVERED Technical
7. AUTHOR(s) Elliot Noma and Janice Johnson		6. PERFORMING ORG. REPORT NUMBER Technical Report No. 62
9. PERFORMING ORGANIZATION NAME AND ADDRESS Human Performance Center University of Michigan Ann Arbor, MI 48104		8. CONTRACT OR GRANT NUMBER(s) N0014-76-C-0648
11. CONTROLLING OFFICE NAME AND ADDRESS Engineering Psychology Program Office of Naval Research 800 N. Quincy, Arlington, VA 22217		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR 197-035
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE June 1979
		13. NUMBER OF PAGES 27
		15. SECURITY CLASS. (of this report) Unclassified
16. DISTRIBUTION STATEMENT (of this Report) Qualified requests may obtain copies of this report from DDC. Others may obtain copies of this report from Office of Technical Services, Department of Commerce.		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		DISTRIBUTION STATEMENT A Approved for public release; Distribution Unlimited
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) multidimensional scaling interpretation of scaling outputs goodness-of-fit measures model testing		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) One of the most crucial aspects of most multidimensional scaling procedures is that the lowest-stress configuration is the output produced. Unfortunately, little is known about the uniqueness of a configuration generated from fallible data, yet this affects the interpretation of the spatial output. It is possible, however, to examine the uniqueness of a scaled solution by constraining the configuration to conform to a particular psychological model. A multidimensional scaling program, CONSCAL, which will allow the		

20. (cont'd)

imposition of such constraints, is proposed. The implications of this approach for interpreting scaling outputs and for model testing in general are discussed. ←



# Constrained Nonmetric Multidimensional Scaling<sup>1</sup>

Elliot Noma

Janice Johnson

The University of Michigan

March 1979

Accession For	
NTIS GRA&I	<input checked="checked" type="checkbox"/>
DDC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By _____	
Distribution/ _____	
Availability Codes	
Dist.	Avail and/or special
A	

### Abstract

One of the most crucial aspects of most multidimensional scaling procedures is that the lowest-stress configuration is the output produced. Unfortunately, little is known about the uniqueness of a configuration generated from fallible data, yet this affects the interpretation of the spatial output. It is possible, however, to examine the uniqueness of a scaled solution by constraining the configuration to conform to a particular psychological model. A multidimensional scaling program, CONSCAL, which will allow the imposition of such constraints, is proposed. The implications of this approach for interpreting scaling outputs and for model testing in general are discussed.



## Constrained Nonmetric Multidimensional Scaling

Multidimensional scaling algorithms yield spatial representations in which the order of the scaled interstimulus distances matches, as closely as possible, the order of observed interstimulus dissimilarities. In most multidimensional scaling programs, the criterion for "as closely as possible" is the minimization of stress or some other index of goodness-of-fit (Kruskal, 1964a). The major goal of all scaling, however, is to reveal latent structure in data. Therefore, interpretability of scaled configurations is the paramount consideration. By the criterion of interpretability, a procedure that only minimizes stress may be inadequate since a minimum-stress configuration may not be the most meaningful or interpretable. This is especially true for nonmetric multidimensional scaling, in which statistical guidelines are generally *ad hoc*.

In some instances it is possible to enhance the interpretability of a minimum-stress configuration by systematically altering it. Though there will often be a concomitant increase in stress, selection of the more interpretable rather than the minimum-stress configuration may be justified.

We propose a method for nonmetric multidimensional scaling called CONSCAL, which constrains a configuration to satisfy a prespecified interpretation. This permits a comparison of the stress value obtained in this way with the minimum stress value (obtained by an unconstrained scaling). Such a comparison provides some indication of how well the prespecified interpretation characterizes the data.

### Constrained Multidimensional Scaling (CONSCAL)

Interpreting each dimension of a multidimensional configuration is conventionally done by comparing the obtained ordering of stimuli along that dimension to unidimensional scale values for those stimuli. These theoretically- or experimentally-derived scale values may also be used to constrain the

configuration. This is important, since a configuration satisfies an interpretation whenever the ordering along a dimension exactly matches the accompanying scale values. Therefore, when the coordinates of the points are constrained, an interpretation is forced upon a configuration. In CONSCAL, the coordinates may be constrained in a specified order along one or more dimensions. All configurations satisfying these constraints are called feasible solutions.

One way to constrain a solution is by using an external penalty function. In this method, an iterative stress-minimizing procedure may initially generate a non-feasible configuration. This minimum-stress configuration, however, will be assessed a penalty so that a feasible configuration will tend to be generated in the next iteration.

CONSCAL uses an alternative approach. A non-feasible configuration may be generated during an iteration, but unlike the penalty function method, the coordinates of points in the configuration,  $X$ , are altered to form a feasible solution before the next iteration. The procedure can be implemented by modifying an iterative multidimensional scaling program, such as KYST or MDSCAL (Kruskal, 1964a), to use a two-step procedure: 1) a method such as the gradient method (Jacoby, Kowalik, & Pizzo, 1972) moves the points into a lower-stress configuration, and 2) points are moved to conform to the ordering constraints. The two steps alternate in each iteration until there is no improvement in stress, only alternation between two configurations - one produced by each half of the procedure.

There are many feasible configurations, so it is necessary to specify a function from a non-feasible coordinate matrix,  $X$ , to a feasible coordinate matrix,  $X'$ . We use a function mapping  $X$  into the feasible  $X'$  that minimizes the sums of the squared distances from  $X$ . This is equivalent to finding  $X'_{ik}$  coordinate values that minimize

$$\sum_{i=1}^n (X_{ik} - X'_{ik})^2$$

for all dimensions  $k$ . An algorithm developed by Kruskal (1964b, p. 128; see also van Eeden, 1957; Bartholomew, 1959; Miles, 1961) is used in step two to move the  $X'_{ik}$ 's into a specified order (one producing a feasible  $X'$ ).

Kruskal's monotone regression, as applied to interpoint distances, has two options for resolving ties, known as the primary and secondary approaches. In the primary approach, tied inter-item dissimilarities need not result in equal interpoint distances, while in the secondary approach, equal dissimilarities must result in equal interpoint distances. In CONSCAL, these two options are also available when specifying the monotone order of projection onto the axes: the primary approach is called weak dimensional monotonicity, and the secondary approach is called semi-strong dimensional monotonicity. In both, if the coordinates  $X_{ik}$  on a dimension  $k$  are constrained by scale value  $c_i$ , then the following is required:

$$\text{if } c_i > c_j, \text{ then } X_{ik} > X_{jk}.$$

Weak dimensional monotonicity makes no additional requirements (note that  $c_i = c_j$  does not restrict the ranks of  $X_{ij}$  and  $X_{jk}$ ). Semi-strong dimensional monotonicity makes the stricter requirement that:

$$\text{if } c_i = c_j, \text{ then } X_{ij} = X_{jk}.$$

Semi-strong dimensional monotonicity is usually used for scaling stimuli in a factorial experimental design, since all tied values of the independent variables used to create the factorial design are usually assumed to have the same coordinate values. However, when hypothesized psychological variables specify the order of projection onto the axes, weak dimensional monotonicity should usually be used. (For example, when two stimuli elicit category estimates of 6 on a 1-to-10 scale, there is little reason to believe that they are psychologically equivalent.)

The CONSCAL program also permits the testing of one nonlinear-constraint model in particular - a radex model. In this model (Levy & Guttman, 1975), the relative location of each point is constrained according to ordered distance



from an origin and ordered angular displacement from an arbitrary vector starting at the origin. Since there are two relevant rank orders that locate each point in a polar coordinate two-dimensional subspace, the CONSCAL program uses the Kruskal monotone regression on both orders to obtain a feasible configuration.

#### An Application: Multidimensional Scaling of Ellipses

The following examples come from a study of the interactions among dimensions of stimulus variation in the perception of ellipses (for a theoretical discussion see Pachella, Somers, and Hardzinski, in press). We were interested in the ability of the following dimension pairs to characterize the judged similarities: physical area and physical eccentricity, judged area and judged eccentricity, or judged length of major and minor axes.

A factorial design with four equally spaced levels of area crossed with four equally spaced levels of eccentricity was employed in constructing the stimuli. The area of the largest ellipse was in a 3:1 ratio to the smallest, and the eccentricity of the most eccentric was in a 1.66:1 ratio to the least eccentric.<sup>2</sup> Black-on-white slides were made of these sixteen ellipses. All ellipses were presented with major axis horizontal.

Four subjects made dissimilarity judgments on a 10-point category scale for all possible pairs of ellipses. The entire set was presented three times, in a different random order each time, and the judgments were averaged for each subject. In another session, subjects made category estimates, on the same 1-10 scale, of the following properties of each ellipse: area, eccentricity, length of major axis, and length of minor axis. The order of these four tasks varied among subjects. Six judgments were made of the four properties for each of the 16 ellipses (384 judgments total), and the judgments were averaged for each subject in each task.

Unconstrained multidimensional scaling of the dissimilarity judgments showed

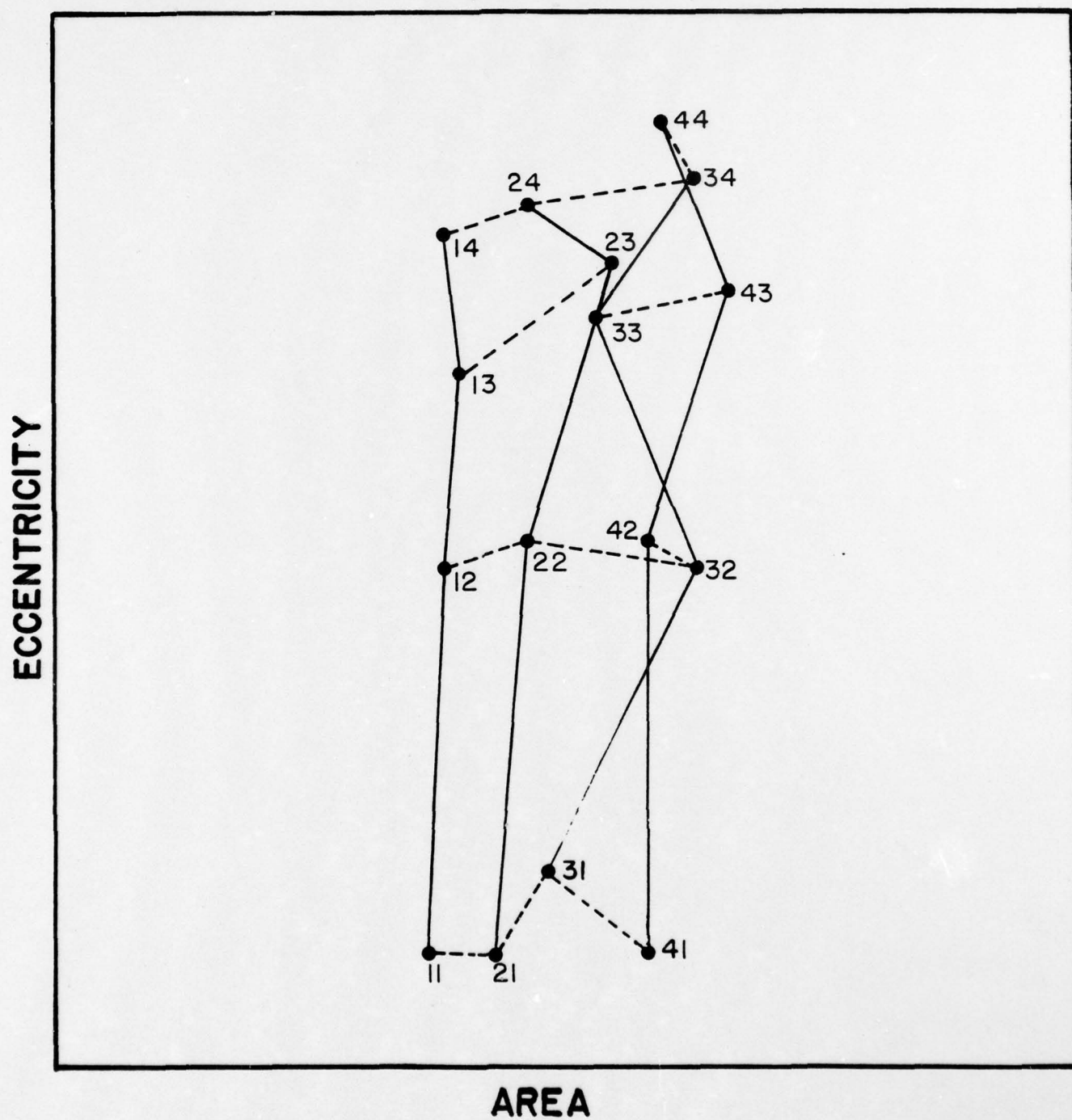


Figure 1. Dimensional interpretation of unconstrained MDS plot, subject RR.



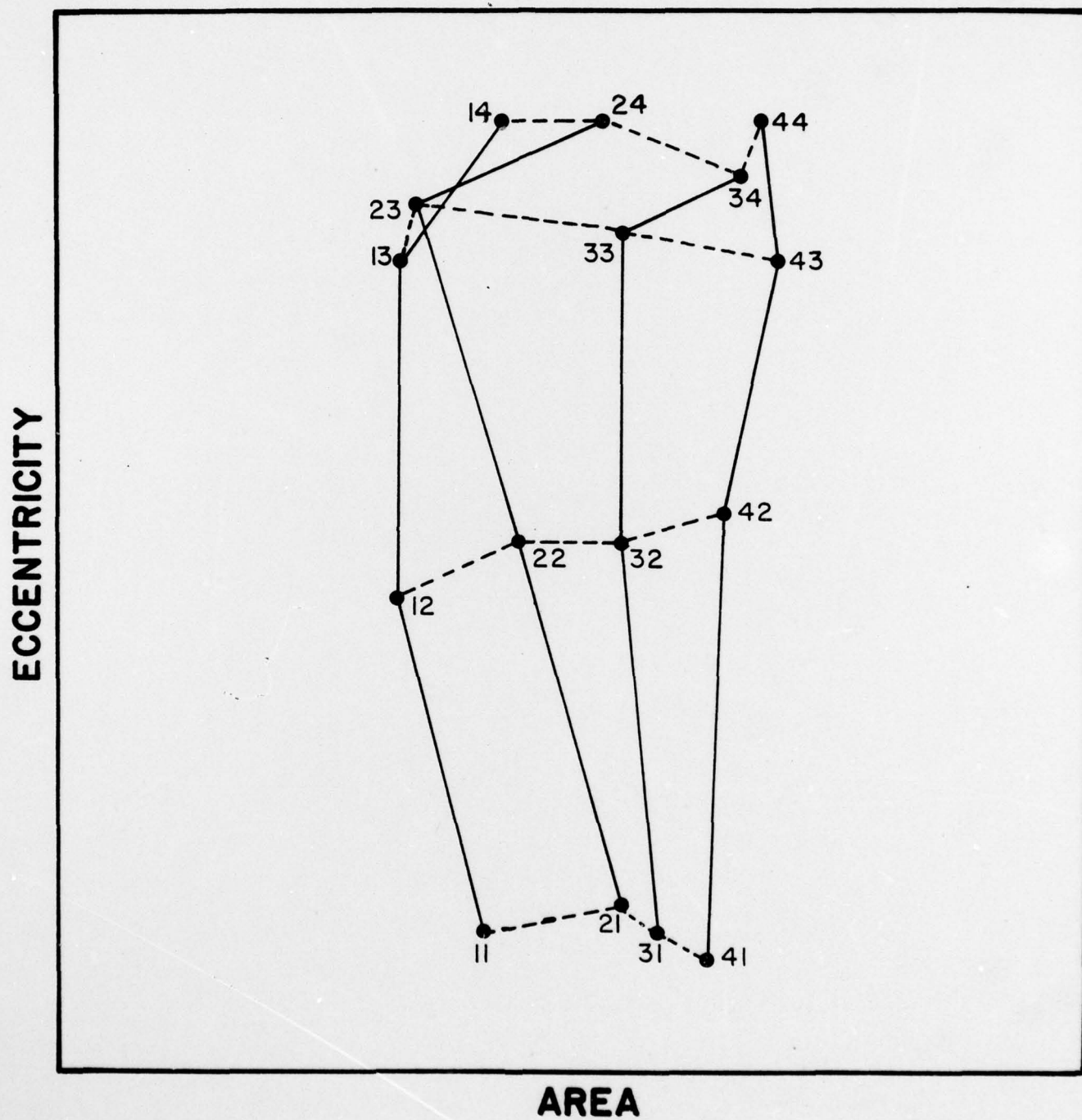


Figure 2. Deminsional interpretation of unconstrained MDS plot, subject JL.

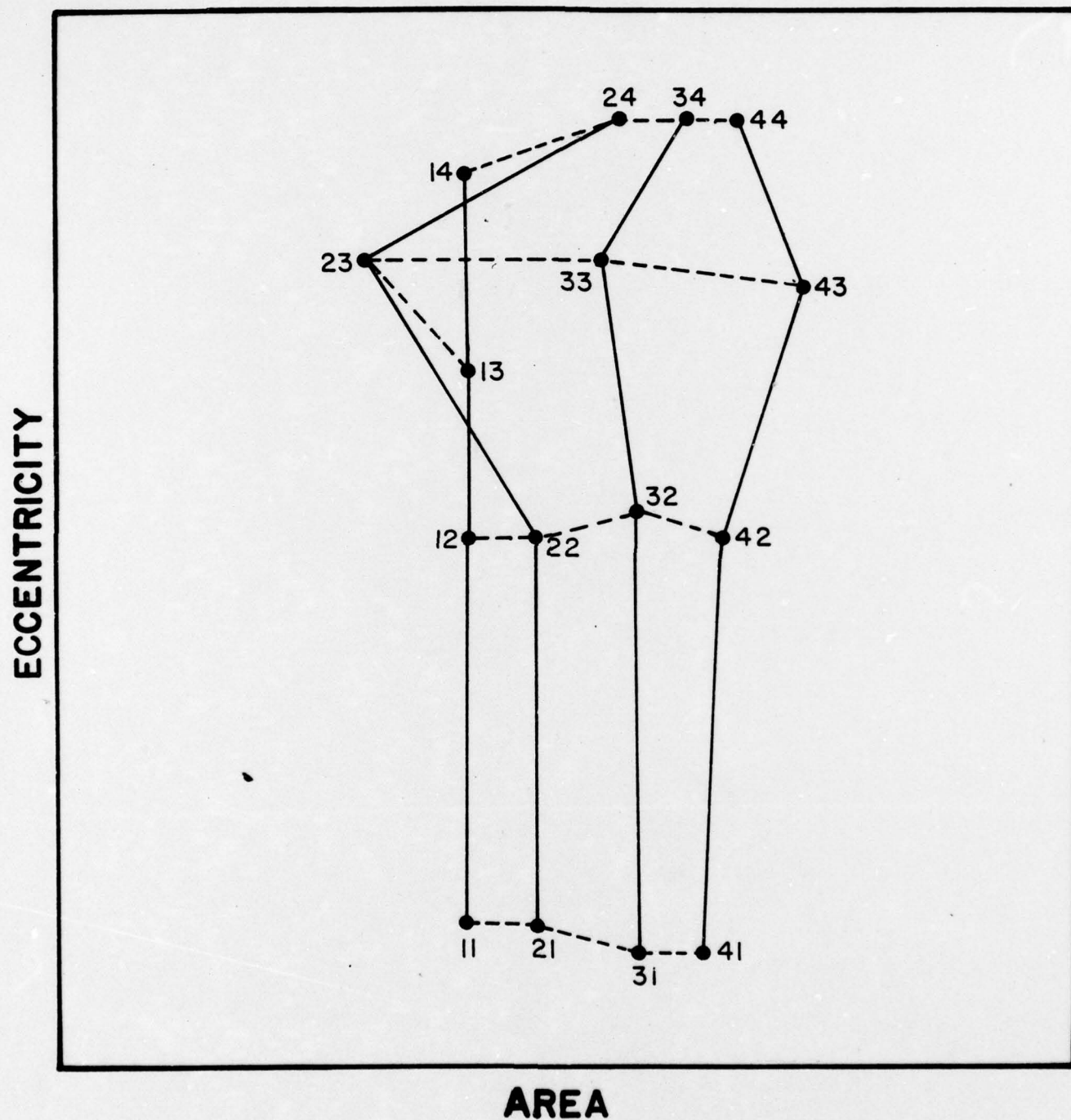


Figure 3. Dimensional interpretation of unconstrained MDS plot, subject TM.



JUDGED ECCENTRICITY

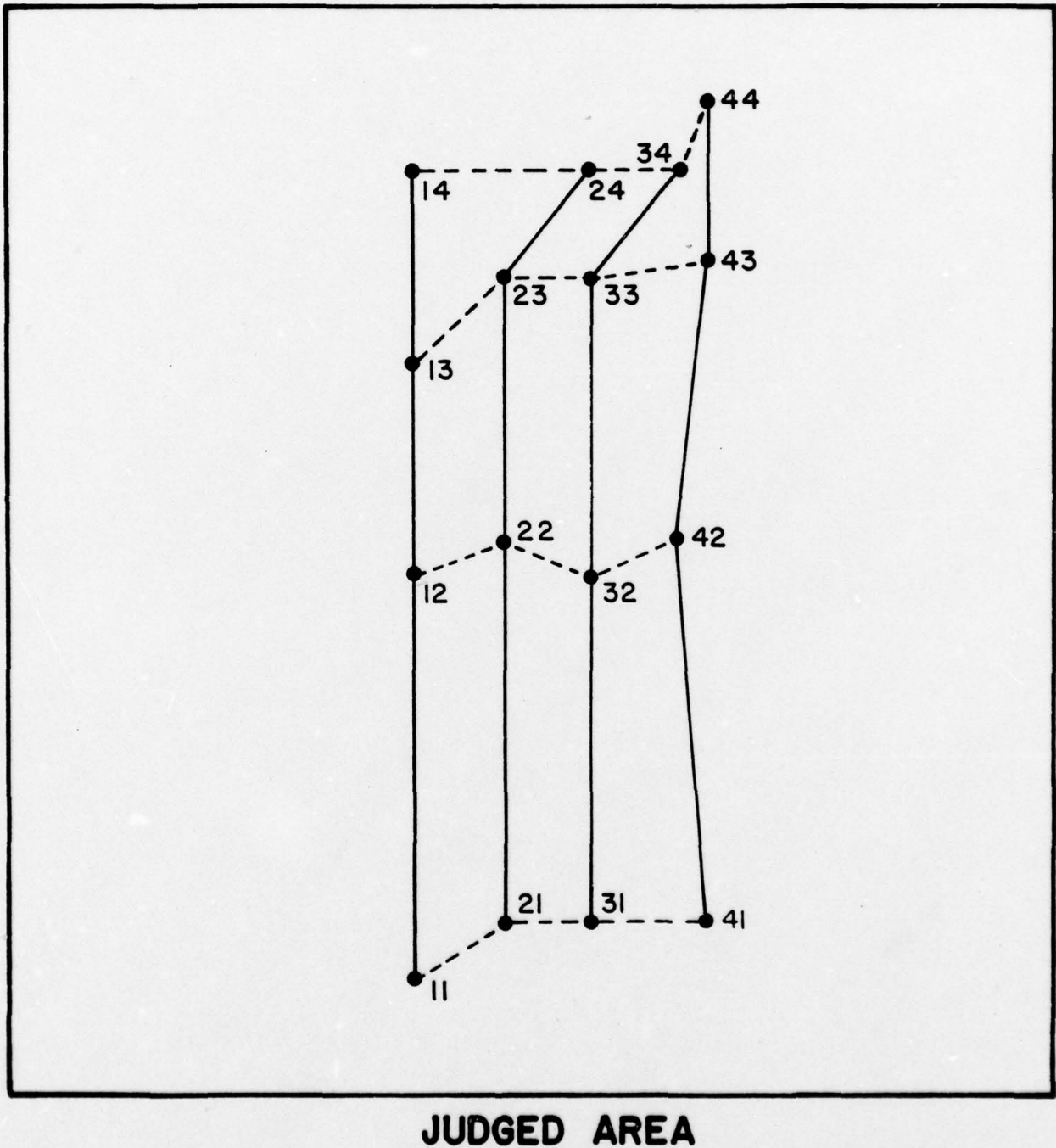


Figure 5. Confirmatory MDS plot; RR. Semi-strong monotonicity: dimensions of judged area and judged eccentricity.



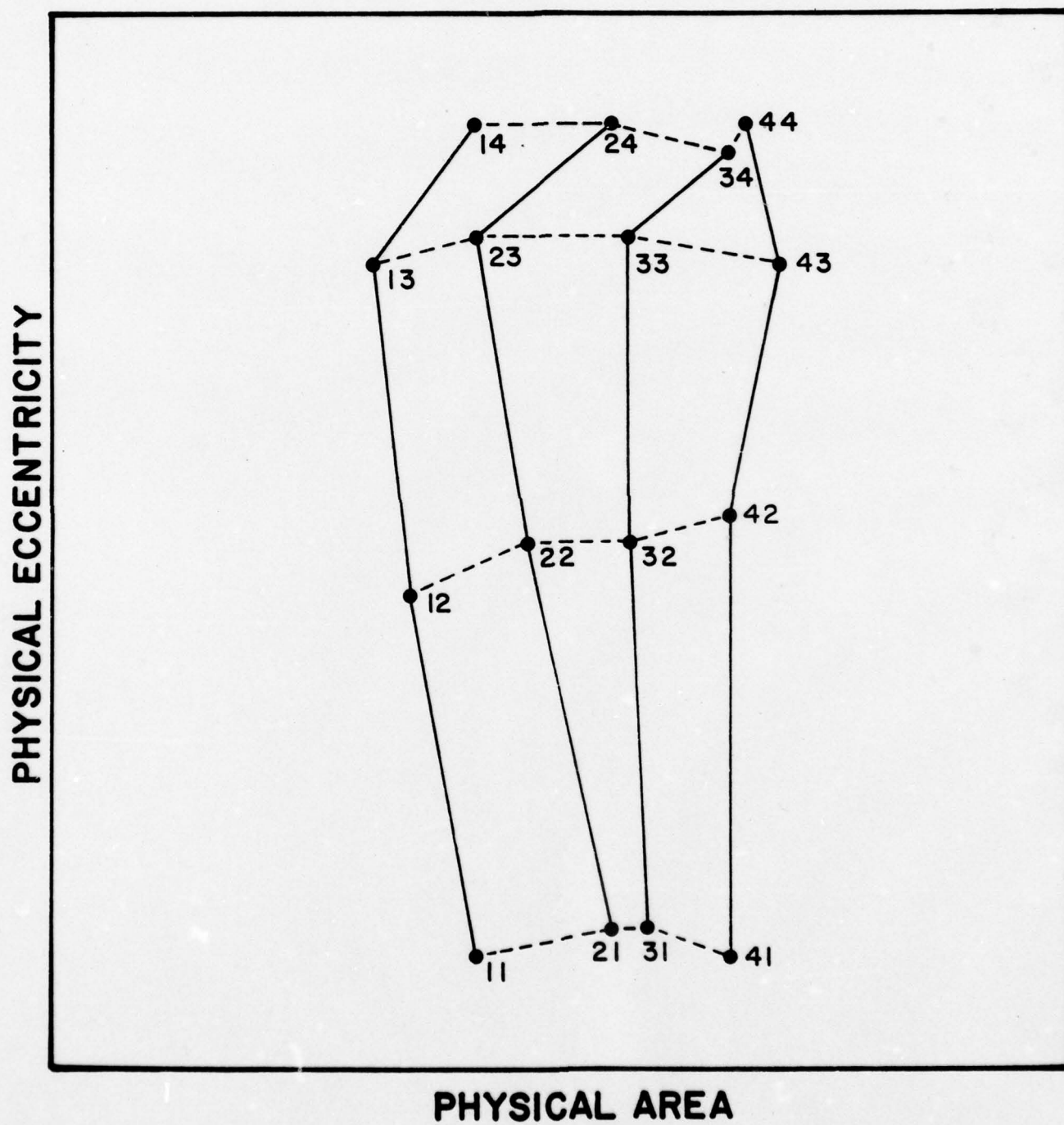
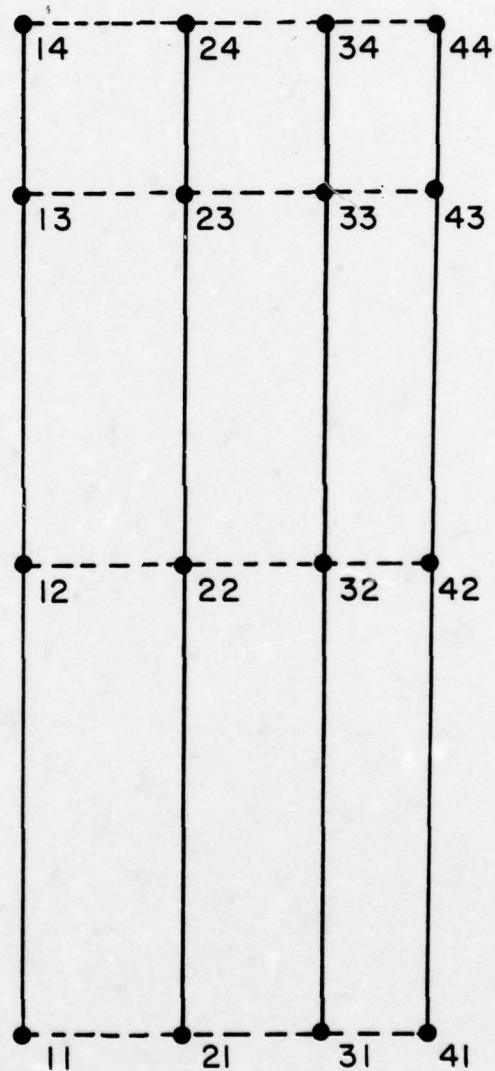


Figure 6. Confirmatory MDS plot; JL. Weak monotonicity: dimensions of physical area and physical eccentricity.



PHYSICAL ECCENTRICITY



PHYSICAL AREA

Figure 7. Confirmatory MDS plot; JL. Semi-strong monotonicity: dimensions of physical area and physical eccentricity.

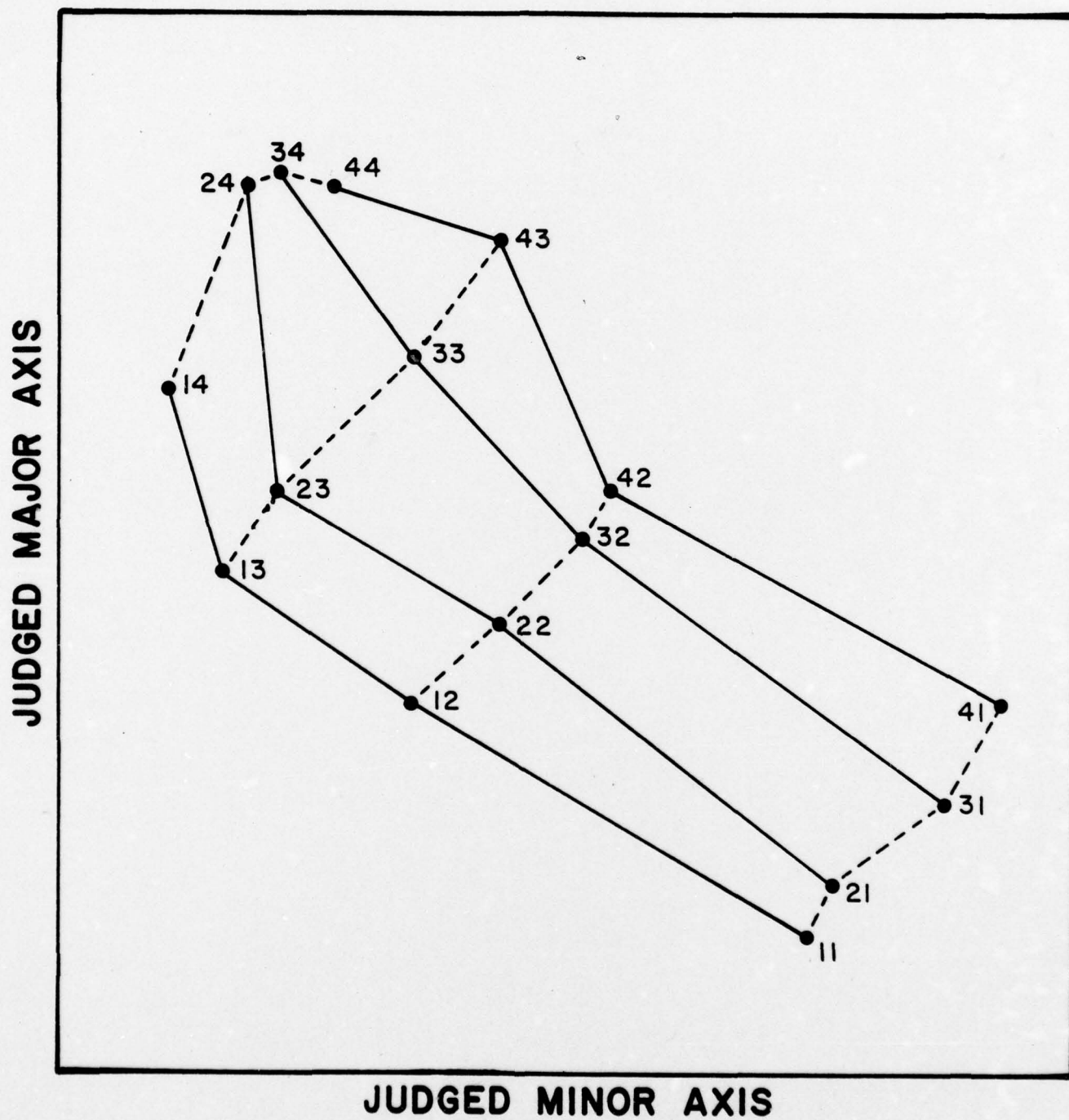


Figure 8. Confirmatory MDS plot; TM. Semi-strong monotonicity: dimensions of judged major and judged minor axes.

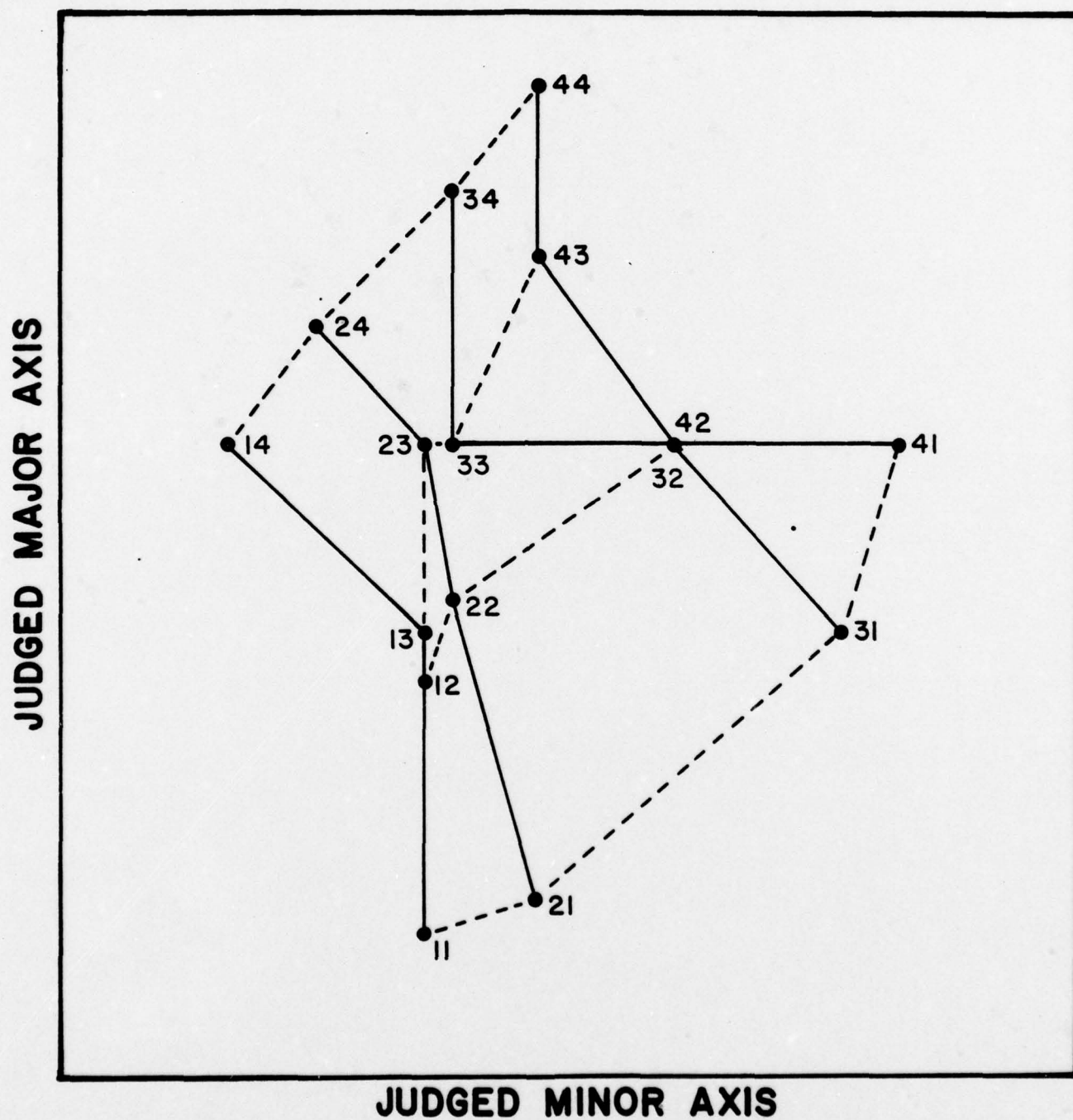
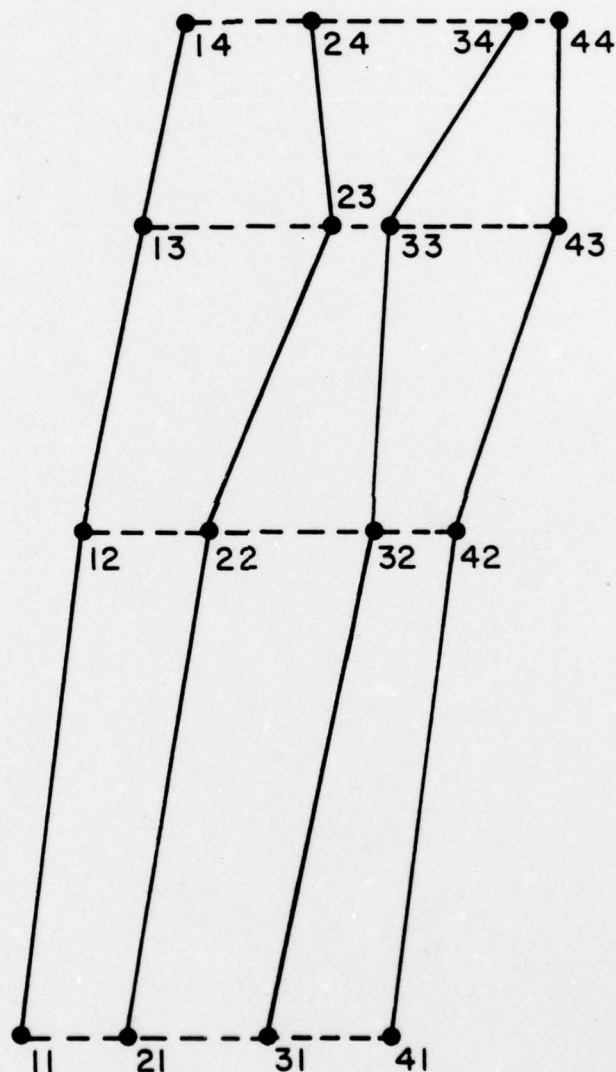


Figure 9. Confirmatory MDS plot; RR. Semi-strong monotonicity: dimensions of judged major and judged minor axes.

PHYSICAL ECCENTRICITY



AREA

Figure 10. Confirmatory MDS plot; RR. Semi-strong monotonicity, with respect to dimension of physical eccentricity only. (Second dimension interpreted as area.)



Table 1  
Stress Values for Configurations with and  
without Constraints, for Four Subjects

Confirmatory	<u>DT</u>	<u>RR</u>	<u>JL</u>	<u>TM</u>
A. Unconstrained	.131	.056	.087	.070
B. Weak Physical Area Physical Eccentricity	.136	.058	.089	.074
C. Semi-Strong Physical Area Physical Eccentricity	.155	.095	.109	.096
D. Weak Psychological Area Psychological Eccentricity	.163	.072	.093	.082
E. Semi-Strong Psychological Area Psychological Eccentricity	.174	.076	.100	.085
F. Weak Psychological Major Axis Psychological Minor Axis	.167	.186	.107	.089
G. Semi-Strong Psychological Major Axis Psychological Minor Axis	.174	.218	.113	.090
H. Semi-Strong Physical Eccentricity	.145	.081	.096	.079



generally good fits for all four of the subjects in two-dimensional Euclidean space. One configuration (subject DT) had a stress of .131 and the other subjects' configurations had stresses ranging from .056 to .087. We were reasonably confident that local minimum problems were being avoided because starts from either random or "hypothesized best fit" (area by eccentricity factorial design) configurations resulted in virtually identical stress values and configurations. For two subjects, a third dimension was added, but this made little difference in stress, and the extra dimension was uninterpretable.

In all four cases, clearly interpretable dimensions of area and eccentricity were present. There were a few minor deviations from the hypothesized orderings along the dimensions, as can be seen in Figures 1-4, and one major reversal of area levels within the smallest eccentricity level in the highest-stress configuration (DT, Figure 4). One question that cannot be answered using traditional stress-minimizing techniques is, how meaningful are such reversals?

-----  
 Insert Figures 1-10 about here  
 -----

Are they merely noise, or does the subject actually have some anomaly in his or her cognitive structure? One way we can try to answer this is to use a constrained multidimensional scaling analysis.

As can be seen in Table 1, constraining the configuration to fit the factorial design according to which the stimuli were constructed causes increases in stress from about .02-.04 for each subject, indicating that this model does

-----  
 Insert Table 1 about here  
 -----

reasonably well for all four subjects. In fact, the configuration with the major reversal (DT, Figure 4) shows the second-lowest increase in stress--only .026. Even without a statistical analysis, this would seem to indicate that even though her deviations from the model appeared to be more systematic than those of the

other subjects, they seem to be no more important.

Comparing judged area and judged eccentricity with physical area and physical eccentricity produced little difference in either stress values (see Table 1) or configurations (see Figures 5 and 6). One would naturally expect the factorial design, with strong monotonicity (see Figure 7), to produce higher stress than any of the other models because of the large number of ties which must be satisfied. These results indicate two things: (1) subjects' scaling of area and eccentricity are reasonably veridical (which is not particularly surprising), and (2) models based upon dimensional combinations of physical versus judged area and eccentricity are for the most part interchangeable, with preference perhaps going for the factorial design model because of its greater simplicity.

Comparing judged area-eccentricity to judged major axis-minor axis models proved more interesting. For three of the subjects, the area-eccentricity and major axis-minor axis models were approximately equivalent in terms of stress, and produced highly similar configurations (compare Figures 3 and 8, for example). However, for one subject, there was a dramatic difference in stress between area-eccentricity and major axis-minor axis configurations. For RR, at least, even though the two models are physically equivalent, they are not psychologically equivalent (compare Figures 5 and 9). This comparison also shows that there can be dramatic individual differences between subjects regarding the applicability of certain models even though the configurations may appear quite similar.

Using confirmatory multidimensional scaling, it is also possible to constrain only a subset of the dimensions. This might be especially helpful if one has strong hypotheses only about some of the dimensions a subject is expected to use, but not about all of them. For example, in Figure 10, eccentricity, but not area, is constrained.

### Discussion

There is no universally accepted procedure for statistically evaluating the stress value of a configuration produced by an unconstrained multidimensional scaling algorithm. The problem of evaluating the difference in stress between constrained and unconstrained configurations is even more complicated. Young (1970) has suggested a degrees-of-freedom approach. Using Young's terminology, in the unconstrained multidimensional scaling of  $N$  points in a space of  $d$  dimensions, there are  $N(N-1)/2$  degrees of freedom of the dissimilarities and  $d(N-1) - [d(d-1)/2]$  degrees of freedom of the coordinates. Young demonstrates that in general, the stress increases with either increases in the degrees of freedom of the dissimilarities (number of points) or decreases in the number of degrees of freedom of the coordinates.

In certain cases, such as that of semi-strong dimensional monotonicity with a factorial design, the degrees of freedom of the coordinates are drastically decreased. For instance, in a four-by-four factorial experimental design, there are 120, or  $16(16-1)/2$ , degrees of freedom of the dissimilarities. In an unconstrained multidimensional scaling of the points in two dimensions there are 29, or  $2(16-1) - [2(2-1)/2]$ , degrees of freedom of the coordinates. By contrast, a constrained multidimensional scaling in a two-dimensional four-by-four design using semi-strong dimensional monotonicity has only 5, or  $2(4-1) - [2(2-1)/2]$ , degrees of freedom of the coordinates. Extending Young's analysis, it might be expected that the stress in the constrained analysis should be much higher than that of the unconstrained solution. However, in our analysis of three of the four subjects we found no large differences in stress when comparing unconstrained and constrained analyses. This seems to imply that the factorial design is the best representation of the data.

There are several reasons why the above approach is inadequate. One problem is that the ordinal-scale assumption of the dissimilarities does not lend itself



to a degrees-of-freedom analysis. Another is that we lack prior knowledge of the number of parameters needed to characterize a constrained solution. It is also unclear how weak dimensional monotonicity and nonfactorial designs could be interpreted in light of a degrees-of-freedom analysis. A further problem is that there are no adequate statistics for evaluating stress for constrained, or unconstrained, multidimensional scaling outputs. This, of course, is a problem for multidimensional scaling in general.

Such difficulties notwithstanding, constrained multidimensional scaling offers unique advantages in its new approach to interpretation. Some such advantages can be seen by comparing and contrasting other interpretation methods with constrained scaling. Of particular interest in this context are other methods for fixing vectors through the space, such as principal components analyses, regression methods, and some methods for drawing cross-configuration comparisons. These interpretation methods fix vectors through the space with an accompanying goodness-of-fit measure after a multidimensional scaling algorithm fixes points in a space and computes the stress. (For a broader discussion of alternative interpretation methods, see Noma and Johnson, 1977).

An example of a method for comparing configurations is PINDIS (Lingoes & Borg, 1978), which fixes axes through a space by combining configurations across subjects. The PINDIS method optimizes two goodness-of-fit criteria - one within each individually scaled configuration (stress), and another across configurations. Such a method is potentially susceptible to tradeoffs between these two criteria. The validity of approaches in which one or more configurations are compared may also be questioned because there might be slight modifications of each configuration that would produce vastly different goodness-of-fit measures across configurations, and change the group space.

Both principal components analysis (see Napior, 1972) and regression of independent variables onto the point coordinates (see Chipman and Carey, 1975)



locate vectors in a fixed space. In both, there are two goodness-of-fit measures optimized by the scaling methods. The multidimensional scaling algorithm minimizes stress, while the principal components and regression methods maximize explained variance. There may be a tradeoff between these two optimization criteria. A configuration with higher than minimal stress may give rise to a better-fitting vector through the space. Alternatively, using a lower-stress configuration in a higher dimensionality might change the fit of the vector. CONSCAL resolves these tradeoff problems by perfectly fitting the vector through space before constructing the configuration.

One other method for testing an interpretation of a configuration may be the Krantz and Tversky (1975) axiomatic tests incorporating an error theory. Such tests set limits on the number of axiom violations acceptable, given a model of random errors. Constrained scaling may also have an advantage over such axiomatic tests in that estimates can be made of the "importance" of violations of the axioms. In other words, some violations of the necessary axioms may be of little psychological interest, since they are an artifact of a particular experimental paradigm. If a subject, for instance, rank orders the inter-stimulus dissimilarities, he may use an arbitrary rule to break ties. However, a measurement theoretic analysis of such data may result in interpreting this bias as an important psychological effect. Scaling these data with a constrained multidimensional scaling shows these anomalies to be unimportant, as they contribute little to the stress.

Using constrained scaling, models of the psychological attributes determining a set of responses may be developed and tested by first scaling the dissimilarity measures using an unconstrained multidimensional scaling method. From this output configuration, and from theoretical arguments, possible interpretations can then be formulated. By applying constrained multidimensional scaling, the relative validity of each interpretation may be assayed.

Despite its shortcomings, CONSCAL offers a different approach to multi-dimensional scaling by emphasizing the testing and comparing of interpretations. By permitting a hypothesis-testing approach, CONSCAL may provide strong support for a particular interpretation of spatially scaled data since it is not vulnerable to stress-interpretability tradeoff problems. In contrast, conventional multi-dimensional scaling approaches are exploratory and provide weaker support for specific interpretations. In summary, CONSCAL determines how the goodness-of-fit measure is affected when a given model is satisfied, rather than determining how closely the scaled output resembles a hypothesized space.

## Footnotes

<sup>1</sup>This research was supported by the Office of Naval Research, Department of Defense, under Contract No. N0014-76-0648 with the Human Performance Center, Department of Psychology, University of Michigan. The senior author was supported by a training grant from NIGMS (GM-01231) to the University of Michigan.

$$\begin{aligned} \text{area} &= \pi \cdot \text{major} \cdot \text{minor} \\ \text{eccentricity} &= \sqrt{1 - \left(\frac{\text{minor}}{\text{major}}\right)^2} \end{aligned}$$

The area levels were (in arbitrary units): .3, .5, .7, .9.

The eccentricity levels were: .600, .940, .986, .995.



## References

- Bartholomew, D. J. A test of homogeneity for ordered alternatives. Biometrika, 1959, 46, 36-48.
- Chipman, S. F., & Carey, S. Anatomy of a stimulus domain: The relation between multidimensional and unidimensional scaling of noise bands. Perception and Psychophysics, 1975, 17, 417-424.
- Jacoby, S. L. S., Kowalik, J. S., & Pizzo, J. T. Iterative methods for nonlinear optimization problems. Englewood Cliffs, NJ: Prentice-Hall, 1972.
- Krantz, D. H., & Tversky, A. Similarity of rectangles: An analysis of subjective dimensions. Journal of Mathematical Psychology, 1975, 12, 4-34.
- Kruskal, J. B. Multidimensional scaling by optimizing goodness of fit to a nonmetric hypothesis. Psychometrika, 1964a, 29, 1-27.
- Kruskal, J. B. Multidimensional scaling: A numerical method. Psychometrika, 1964b, 29, 115-129.
- Levy, S., & Guttman, L. On the multivariate structure of wellbeing. Social Indicators Research, 1975, 2, 361-388.
- Lingoes, J. C., & Borg, I. A direct approach to individual differences scaling using increasingly complex transformations. Psychometrika, 1978, 43, 491-520.
- Miles, R. E. The complete amalgamation into blocks, by weighted means, of a finite set of real numbers. Biometrika, 1959, 46, 317-327.
- Napier, D. Nonmetric multidimensional techniques for summated ratings. In R. N. Shepard, A. K. Romney, & S. B. Nerlove (Eds.), Multidimensional Scaling, Vol. I. New York: Seminar, 1972.
- Noma, E., & Johnson, J. Constraining nonmetric multidimensional scaling configurations. Human Performance Center Technical Report No. 60. Ann Arbor: The University of Michigan, August 1977.
- Pachella, R. G., Somers, P., & Hardzinski, M. Psychophysical compatibility and



the perception of multidimensional displays. In D. Getty and J. Howard (Eds.), Auditory pattern recognition. New York: Lawrence Erlbaum Associates, in press.

van Eeden, C. Maximum likelihood estimation of partially or completely ordered parameters. Proc. Akademie van Wetenschappen, Series A, 1957, 60, 128-136.

Young, F. W. Nonmetric multidimensional scaling: Recovery of metric information. Psychometrika, 1970, 35, 455-473.

OFFICE OF NAVAL RESEARCH

Code 455

TECHNICAL REPORTS DISTRIBUTION LIST

OSD

CDR Paul R. Chatelier  
Military Assistant for Training  
and Personnel Technology  
Office of the Deputy Under Secretary  
of Defense  
OUSDRE (E&LS)  
Pentagon, Room 3D129  
Washington, DC 20301

Commanding Officer  
ONR Branch Office  
ATTN: Dr. J. Lester  
Building 114, Section D  
666 Summer Street  
Boston, MA 02210

Department of the Navy

Director  
Engineering Psychology Programs  
Code 455  
Office of Naval Research  
800 North Quincy Street  
Arlington, Virginia 22217 (5 cps)

Commanding Officer  
ONR Branch Office  
ATTN: Dr. C. Davis  
536 South Clark Street  
Chicago, IL 60605

Director  
Operations Research Programs  
Code 434  
Office of Naval Research  
800 North Quincy Street  
Arlington, Virginia 22217

Commanding Officer  
ONR Branch Office  
ATTN: Dr. E. Gloye  
1030 East Green Street  
Pasadena, CA 91106

Director  
Statistics and Probability Program  
Code 436  
Office of Naval Research  
800 North Quincy Street  
Arlington, Virginia 22217

Office of Naval Research  
Scientific Liaison Group  
American Embassy, Room A-407  
APO San Francisco, CA 96503

Director  
Physiology Program  
Code 441  
Office of Naval Research  
800 North Quincy Street  
Arlington, Virginia 22217

Director  
Naval Research Laboratory  
Technical Information Division  
Code 2627  
Washington, DC 20375 (6 cps)

Dr. Robert G. Smith  
Office of the Chief of Naval  
Operations, OP987H  
Personnel Logistics Plans  
Washington, DC 20350

Department of the Navy

Naval Training Equipment Center  
ATTN: Technical Library  
Orlando, FL 32813

Human Factors Department  
Code N215  
Naval Training Equipment Center  
Orlando, FL 32813

Dr. Alfred F. Smode  
Training Analysis and Evaluation Group  
Naval Training Equipment Center  
Code N-00T  
Orlando, FL 32813

Dr. Gary Poock  
Operations Research Department  
Naval Postgraduate School  
Monterey, CA 93940

Dean of Research Administration  
Naval Postgraduate School  
Monterey, CA 93940

Mr. Warren Lewis  
Human Engineering Branch  
Code 8231  
Naval Ocean Systems Center  
San Diego, CA 92152

Dr. A. L. Slafkosky  
Scientific Advisor  
Commandant of the Marine Corps  
Code RD-1  
Washington, DC 20380

Mr. Arnold Rubinstein  
Naval Material Command  
NAVMAT 98T24  
Washington, DC 20360

Commander  
Naval Air Systems Command  
Human Factors Programs  
NAVAIR 340F  
Washington, DC 20361

Commander  
Naval Air Systems Command  
Crew Station Design,  
NAVAIR 5313  
Washington, DC 20361

Dr. James Curtin  
Naval Sea Systems Command  
Personnel & Training Analyses Office  
NAVSEA 074C1  
Washington, DC 20362

Commander  
Naval Electronics Systems Command  
Human Factors Engineering Branch  
Code 4701  
Washington, DC 20360

Bureau of Naval Personnel  
Special Assistant for Research Liaison  
PERS-OR  
Washington, DC 20370

CDR R. Gibson  
Bureau of Medicine and Surgery  
Aerospace Psychology Branch  
Code 513  
Washington, DC 20372

LCDR Robert Biersner  
Naval Medical R&D Command  
Code 44  
Naval Medical Center  
Bethesda, MD 20014



Department of the Navy

Dr. Arthur Bachrach  
Behavioral Sciences Department  
Naval Medical Research Institute  
Bethesda, MD 20014

LCDR T. Berghage  
Naval Medical Research Institute  
Behavioral Sciences Department  
Bethesda, MD 20014

Dr. George Moeller  
Human Factors Engineering Branch  
Submarine Medical Research Lab  
Naval Submarine Base  
Groton, CT 06340

Chief  
Aerospace Psychology Division  
Naval Aerospace Medical Institute  
Pensacola, FL 32512

Dr. Fred Muckler  
Navy Personnel Research and  
Development Center  
Manned Systems Design, Code 311  
San Diego, CA 92152

Navy Personnel Research and  
Development Center  
Management Support Department  
Code 210  
San Diego, CA 92152

Navy Personnel Research and  
Development Center  
Code 305  
San Diego, CA 92152

CDR P. M. Curran  
Human Factors Engineering Division  
Naval Air Development Center  
Warminster, PA 18974

Department of the Army

Mr. J. Barber  
HQS, Department of the Army  
DAFE-PBR  
Washington, DC 20546

Dr. Joseph Zeidner  
Technical Director  
U.S. Army Research Institute  
5001 Eisenhower Avenue  
Alexandria, VA 22333

Director, Organizations and Systems  
Research Laboratory  
U.S. Army Research Institute  
5001 Eisenhower Avenue  
Alexandria, VA 22333

Technical Director  
U.S. Army Human Engineering Labs  
Aberdeen Proving Ground, MD 21005

Department of the Air Force

U.S. Air Force Office of  
Scientific Research  
Life Sciences Directorate, NL  
Bolling Air Force Base  
Washington, DC 20332

Air University Library  
Maxwell Air Force Base, AL 36112

Other Government Agencies

Defense Documentation Center  
Cameron Station, Bldg. 5  
Alexandria, VA 22314 (12 cps)



Other Government Agencies

Dr. Stephen J. Andriole  
Director, Cybernetics Technology Office  
Defense Advanced Research Projects Agency  
1400 Wilson Blvd.  
Arlington, VA 22209

Dr. Stanley Deutsch  
Office of Life Sciences  
National Aeronautics and  
Space Administration  
500 Independence Avenue  
Washington, DC 20546

Other Organizations

Dr. James H. Howard  
Department of Psychology  
Catholic University  
Washington, DC 20064

Journal Supplement Abstract Service  
American Psychological Association  
1200 17th Street, N. W.  
Washington, DC 20036 (3 cps)

Dr. J. A. Swets  
Bolt, Beranek & Newman, Inc.  
50 Moulton Street  
Cambridge, MA 02138

Dr. Robert Williges  
Human Factors Laboratory  
Virginia Polytechnical Institute  
and State University  
130 Whittemore Hall  
Blacksburg, VA 24061

Foreign Addressees

Director, Human Factors Wing  
Defence and Civil Institute  
of Environmental Medicine  
P. O. Box 2000  
Downsville, Toronto, Ontario  
CANADA